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Preface

The availability of computers has revolutionized every field which depends on numerical calculations. The scientist or engineer can be greatly aided in his work if he knows how to enlist the aid of his new ally. The purpose of this book is to help the undergraduate student learn how to apply a computer to many different types of practical problems. The book is written in a manner that should instill enthusiasm in the reader and help convince him that the computer is one of the greatest labor saving devices ever to come along.

This book's philosophy differs from most others written on numerical methods or numerical analysis. In a typical numerical-analysis text, much time is spent on error analysis. However, in practical applications, usually little time is devoted to rigorous error analysis.

Instead of relying solely on error analysis to estimate the accuracy of answers, other methods are emphasized in this text. Observing how a process converges can give insight into its accuracy. Also, solving a problem two different ways can verify a solution. Although error analysis is not very practical as a tool for estimating accuracy, it does have other uses. It can be used to compare different numerical methods and demonstrate that, on the average, one is superior to another. Or a knowledge of error properties can be used to improve a numerical method.

Avoiding a lengthy investigation of error analysis allows time for the reader to become acquainted with optimization techniques. Numerical methods and optimization techniques are intimately related, but unfortunately they are not generally taught in the same course. However, since both numerical methods and optimization techniques are usually iterative procedures, they have a common philosophy and application. In fact, as demonstrated in this book, an optimization technique can be viewed as a collection of numerical methods which have been linked together in a specific way. Thus, once a student has become familiar with numerical methods the extension to optimization techniques is very natural.

This text does not attempt to be a complete catalog of numerical methods or optimization techniques—volumes would be needed for this. For a specific problem, the specialist can probably consult the literature