where $V$ = molar volume
$P$ = absolute pressure
$T$ = absolute temperature
$R$ = universal gas constant

The gas constant $R$ assumes different values for different sets of units. Common values are shown in Table 3-1. For the remainder of this book, unless otherwise noted, pressure will be in bars, volume in $cm^3/mol$, and the term mol will refer to gram moles. Note that 1 bar = $10^5$ Pa = $10^5$ N/$m^2$ and 1 atm = 1.01325 bar. For an ideal gas $Z = 1.0$. For real gases, $Z$ is normally less than 1 except at high reduced temperatures and pressures. Equation (3-2.1) can also be used to define $Z$ for a liquid; in this case $Z$ is normally much less than unity.

The compressibility factor is often correlated with the reduced temperature $T_r$ and pressure $P_r$ as

$$Z = f(T_r, P_r) \qquad (3\text{-}2.2)$$

where $T_r = T/T_c$ and $P_r = P/P_c$. The function $f(\ )$ has been obtained from experimental $PVT$ data by Nelson and Obert [73], and the final curves are shown in Figs. 3-1 to 3-3. Except as noted below, the use of these figures to obtain $Z$ at a given $T_r$ and $P_r$ should lead to errors of less than 4 to 6 percent except near the saturation curve or near the critical point, where $Z$ is very sensitive to both $T_r$ and $P_r$.

Figures 3-1 and 3-3 should not be used for strongly polar fluids, nor are they recommended for helium, hydrogen, or neon unless special, modified critical constants are used [28, 62, 69, 74]. For very high pressures or very high temperatures, the reduced pressure-temperature-density charts of Breedveld and Prausnitz [9] are useful.

Many graphs similar to those in Figs. 3-1 to 3-3 have been published. All differ somewhat, because each reflects the author's choice of experimental data and how the data are smoothed. Those shown are as accurate as any two-parameter plots published, and they have the added advantage that volumes can be found directly. Note, however, that in these figures

**TABLE 3-1  Values of the Gas Constant $R$**

| Value of $R$ | Unit† for $R$ |
|---|---|
| 83.144 | $bar \cdot cm^3/(mol \cdot K)$ |
| 8.3144 | $J/(mol \cdot K)$ |
| 10.732 | $psia \cdot ft^3/(lb\text{-}mol \cdot °R)$ |
| 82.057 | $atm \cdot cm^3/(mol \cdot K)$ |

†The unit mol refers to gram moles.


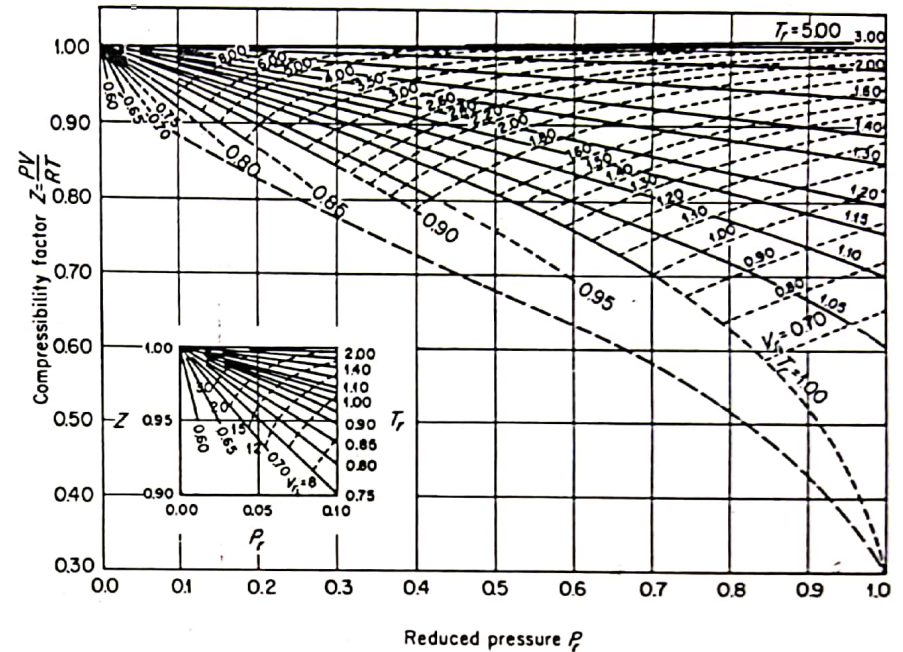
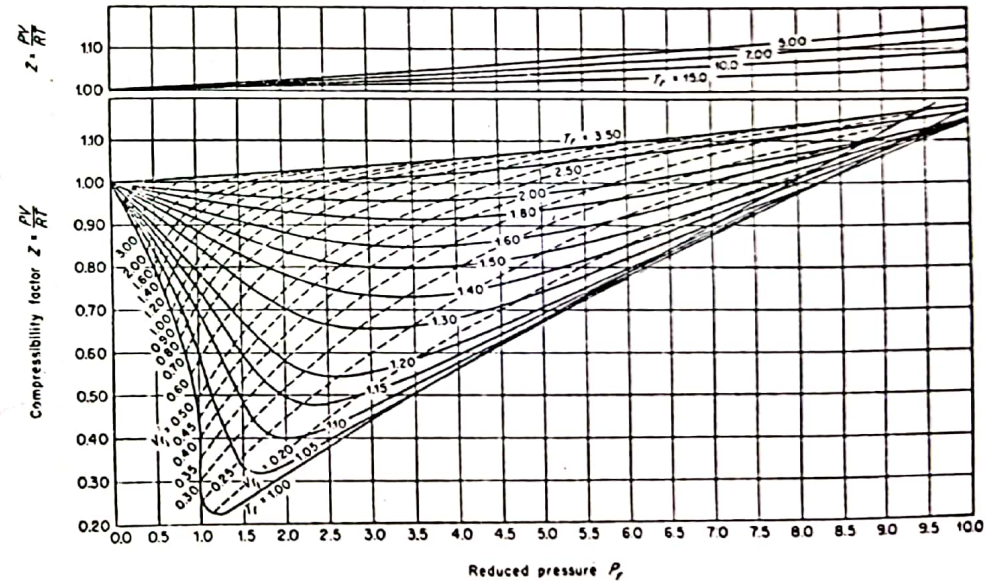Figure 3-1  Generalized compressibility chart. $V_{r_i}$ is $V/(RT_c/P_c)$. (From Ref. 73.)



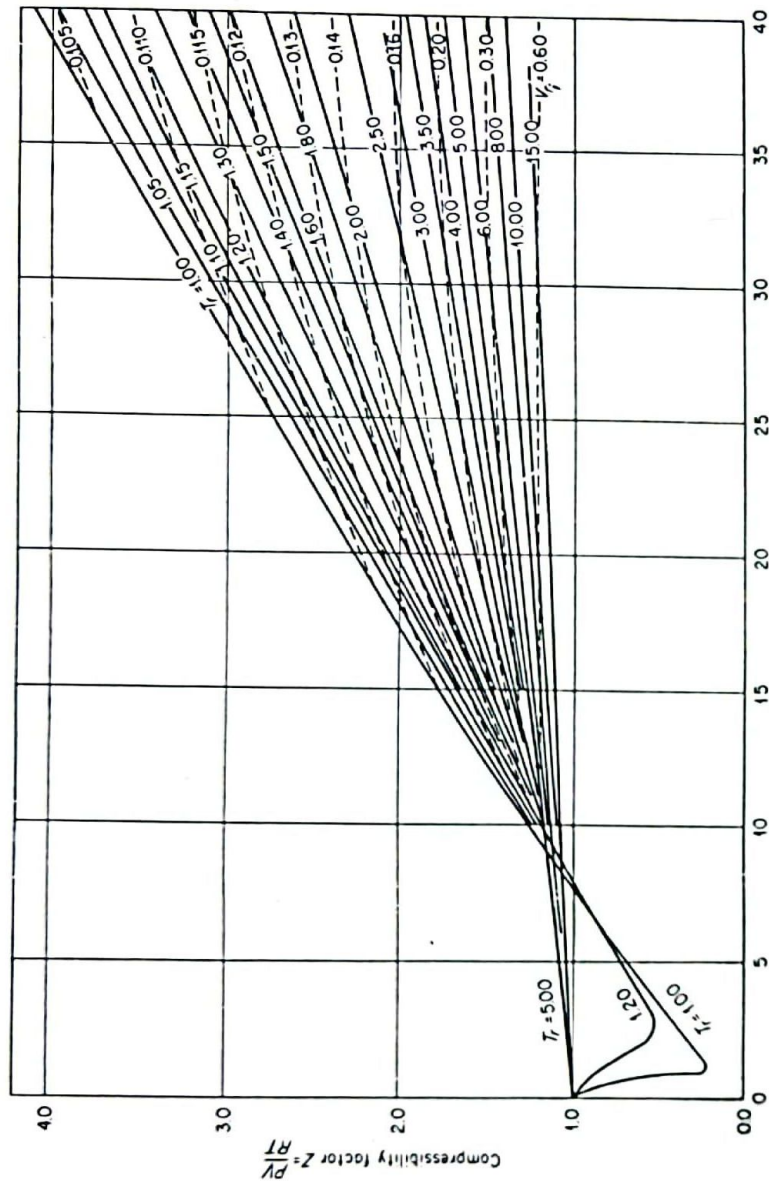Figure 3-2  Generalized compressibility chart. $V_{r_i}$ is $V/(RT_c/P_c)$. (From Ref. 73.)

$V_{r_i}$ is not defined in the usual manner, that is, $V/V_c$, but instead is a "ideal reduced volume" given by

$$V_{r_i} = \frac{V}{RT_c/P_c} \tag{3-2.3}$$

Equation (3-2.2) is an example of the law of corresponding states. This law, though not exact, suggests that reduced properties of all fluids are essentially the same if compared at equal reduced temperatures and pressures. For $PVT$ properties, this law gives

$$V_r = \frac{V}{V_c} = \frac{(Z/Z_c)(T/T_c)}{P/P_c} = f_1(T_r, P_r)$$

or

$$Z = Z_c f_2(T_r, P_r) \tag{3-2.4}$$

Except for monatomic gases, highly polar fluids, and fluids composed of large molecules, values of $Z_c$ for most organic compounds range from 0.27 to 0.29. If it is assumed to be a constant, Eq. (3-2.4) reduces to Eq. (3-2.2). In Sec. 3-3, $Z_c$ is introduced as a third correlating parameter (in addition to $T_c$ and $P_c$) to estimate $Z$, but not in the form of Eq. (3-2.4).

In Eq. (3-2.2), $T_c$ and $P_c$ are scaling factors to reduce $T$ and $P$; i.e., to make them nondimensional. Other scaling factors have been proposed, but none have been widely accepted. A tabulation of $T_c$ and $P_c$ for a number of elements and compounds is given in Appendix A, and methods for estimating them are described in Sec. 2-2.

## 3-3 Three-Parameter Correlations

Equation (3-2.2) is a two-parameter equation of state, the two parameters being $T_c$ and $P_c$. That is, by knowing $T_c$ and $P_c$ for a given fluid, it is possible to estimate the volumetric properties at various temperatures and pressures. The calculation may involve the use of Figs. 3-1 to 3-3, or one may employ an analytical function for $f(\ )$ in Eq. (3-2.2). Both methods are only approximate. Many suggestions which retain the general concept yet allow an increase in accuracy and applicability have been offered. In general, the more successful modifications have involved the inclusion of an additional third parameter into the function expressed by Eq. (3-2.2). Most often, this third parameter is related to the reduced vapor pressure at some specified reduced temperature or to some volumetric property at or near the critical point, although one correlation employs the molar polarizability as the third parameter [93]. Two common and well-tested three-parameter correlations are described below.

Assume that there are different, but unique, functions $Z = f(T_r, P_r)$ for each group of pure substances with the same $Z_c$. Then, for each $Z_c$ we have



**Figure 3-3** Generalized compressibility chart. $V_{r_i}$ is $V/(RT_c/P_c)$. (From Ref. 73.)

a different set of Figs. 3-1 to 3-3. All fluids with the same $Z_c$ values then follow the $Z$-$T_r$-$P_r$ behavior shown on charts drawn for that particular $Z_c$. Such a structuring indeed leads to a significant increase in accuracy. This is exactly what was done in the development of the Lydersen-Greenkorn-Hougen tables, which first appeared in 1955 [53] and were later modified [37]. There $Z$ is tabulated as a function of $T_r$ and $P_r$ with separate tables for various values of $Z_c$. Edwards and Thodos [20] have also utilized $Z_c$ in a correlation to estimate saturated vapor densities of nonpolar compounds.

An alternate third parameter is the Pitzer acentric factor [86–89], defined in Sec. 2-3. This factor is an indicator of the nonsphericity of a molecule's force field; e.g., a value of $\omega = 0$ denotes rare-gas spherical symmetry. Deviations from simple-fluid behavior are evident when $\omega > 0$. Within the context of the present discussion, it is assumed that all molecules with equal acentric factors have identical $Z = f(T_r, P_r)$ functions, as in Eq. (3-2.2). However, rather than prepare separate $Z$, $T_r$, $P_r$ tables for different values of $\omega$, it was suggested that a linear expansion could be employed:

$$Z = Z^{(0)}(T_r, P_r) + \omega Z^{(1)}(T_r, P_r) \tag{3-3.1}$$

Thus, the $Z^{(0)}$ function would apply to spherical molecules, and the $Z^{(1)}$ term is a deviation function.

Pitzer et al. tabulated $Z^{(0)}$ and $Z^{(1)}$ as functions of $T_r$ and $P_r$ [89], and Edmister has shown the same values graphically [18]. Several modifications as well as extensions to wider ranges of $T_r$ and $P_r$ have been published [38, 51, 99]. Tables 3-2 and 3-3 list those prepared by Lee and Kesler [47]. The method of calculation is described in Sec. 3-7. With Tables 3-2 and 3-3, $Z$ can be determined for both gases and liquids.* The $Z^{(0)}$ table agrees well with that presented originally by Pitzer et al. over the range of $T_r$ and $P_r$ common to both. The deviation function table of Lee and Kesler (Table 3-3) differs somewhat from that of Pitzer and Curl, but extensive testing [47, 114] indicates the new table is the more accurate.

Tables 3-2 and 3-3 were not intended to be applicable to strongly polar fluids, though they are often so used with surprising accuracy except at low temperatures near the saturated vapor region. Though none have been widely adopted, special techniques have been suggested to modify Eq. (3-3.1) for polar materials [21, 31, 50, 75, 111, 120].

Considerable emphasis has been placed on the Pitzer-Curl generalized relation. It has proved to be accurate and general when applied to pure gases. Only the acentric factor and critical temperature and pressure need

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*For mixtures, see Table 4.3.

be known. It is probably the most successful and useful result of corresponding states theory [48, 109, 110].

$CCl_2F_2$

**Example 3-1** Estimate the specific volume of dichlorodifluoromethane vapor at 20.67 bar and 366.5 K.

**solution** From Appendix A, $T_c = 385.0$ K, $P_c = 41.4$ bar, and $\omega = 0.204$.

$$T_r = \frac{366.5}{385.0} = 0.952 \qquad P_r = \frac{20.67}{41.4} = 0.499$$

From Fig. 3-1, $Z = 0.77$ and

$$V = \frac{ZRT}{P} = \frac{(0.77)(83.14)(366.5)}{20.67} = 1134 \text{ cm}^3/\text{mol}$$

The value reported in the literature is 1109 cm³/mol [4].

If the Pitzer-Curl method were to be used, from Tables 3-2 and 3-3, $Z^{(0)} = 0.761$ and $Z^{(1)} = -0.082$. From Eq. (3-3.1),

$$Z = 0.761 + (0.204)(-0.082) = 0.744$$

$$V = \frac{ZRT}{P} = 1097 \text{ cm}^3/\text{mol}$$

## 3-4 Analytical Equations of State

An analytical equation of state is an algebraic relation between pressure, temperature, and molar volume. Three classes of equations of state are presented in the next three sections. The virial equation is discussed in Sec. 3-5. In its truncated form, it is a simple equation, and it can represent only modest deviations in the vapor phase from ideal-gas behavior. In Sec. 3-6, equations which are cubic in volume are discussed. These equations can represent both liquid and vapor behavior of nonpolar molecules over limited ranges of temperature and pressure, and they remain relatively simple from a computational point of view. Section 3-7 describes the Lee-Kesler generalized version of the Benedict-Webb-Rubin equation, which is applicable over broader ranges of temperatures and pressure than are the cubic equations. But it is also computationally more complex.

## 3-5 Virial Equation

The virial equation of state is a polynomial series in inverse volume which is explicit in pressure and can be derived from statistical mechanics:

$$P = \frac{RT}{V} + \frac{RTB}{V^2} + \frac{RTC}{V^3} + \cdots \tag{3-5.1}$$

The parameters $B, C, \ldots$ are called the second, third, $\ldots$ virial coefficients and are functions only of temperature for a pure fluid. Much has been written about this particular equation, and several reviews have been pub-

**TABLE 3-2    Values of $Z^{(0)}$**

| $T_r$ | $P_r$ | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.010 | 0.050 | 0.100 | 0.200 | 0.400 | 0.600 | 0.800 | 1.000 | 1.200 | 1.500 | 2.000 | 3.000 | 5.000 | 7.000 | 10.000 |
| 0.30 | 0.0029 | 0.0145 | 0.0290 | 0.0579 | 0.1158 | 0.1737 | 0.2315 | 0.2892 | 0.3470 | 0.4335 | 0.5775 | 0.8648 | 1.4366 | 2.0048 | 2.8507 |
| 0.35 | 0.0026 | 0.0130 | 0.0261 | 0.0522 | 0.1043 | 0.1564 | 0.2084 | 0.2604 | 0.3123 | 0.3901 | 0.5195 | 0.7775 | 1.2902 | 1.7987 | 2.5539 |
| 0.40 | 0.0024 | 0.0119 | 0.0239 | 0.0477 | 0.0953 | 0.1429 | 0.1904 | 0.2379 | 0.2853 | 0.3563 | 0.4744 | 0.7095 | 1.1758 | 1.6373 | 2.3211 |
| 0.45 | 0.0022 | 0.0110 | 0.0221 | 0.0442 | 0.0882 | 0.1322 | 0.1762 | 0.2200 | 0.2638 | 0.3294 | 0.4384 | 0.6551 | 1.0841 | 1.5077 | 2.1338 |
| 0.50 | 0.0021 | 0.0103 | 0.0207 | 0.0413 | 0.0825 | 0.1236 | 0.1647 | 0.2056 | 0.2465 | 0.3077 | 0.4092 | 0.6110 | 1.0094 | 1.4017 | 1.9801 |
| 0.55 | 0.9804 | 0.0098 | 0.0195 | 0.0390 | 0.0778 | 0.1166 | 0.1553 | 0.1939 | 0.2323 | 0.2899 | 0.3853 | 0.5747 | 0.9475 | 1.3137 | 1.8520 |
| 0.60 | 0.9849 | 0.0093 | 0.0186 | 0.0371 | 0.0741 | 0.1109 | 0.1476 | 0.1842 | 0.2207 | 0.2753 | 0.3657 | 0.5446 | 0.8959 | 1.2398 | 1.7440 |
| 0.65 | 0.9881 | 0.9377 | 0.0178 | 0.0356 | 0.0710 | 0.1063 | 0.1415 | 0.1765 | 0.2113 | 0.2634 | 0.3495 | 0.5197 | 0.8526 | 1.1773 | 1.6519 |
| 0.70 | 0.9904 | 0.9504 | 0.8958 | 0.0344 | 0.0687 | 0.1027 | 0.1366 | 0.1703 | 0.2038 | 0.2538 | 0.3364 | 0.4991 | 0.8161 | 1.1241 | 1.5729 |
| 0.75 | 0.9922 | 0.9598 | 0.9165 | 0.0336 | 0.0670 | 0.1001 | 0.1330 | 0.1656 | 0.1981 | 0.2464 | 0.3260 | 0.4823 | 0.7854 | 1.0787 | 1.5047 |
| 0.80 | 0.9935 | 0.9669 | 0.9319 | 0.8539 | 0.0661 | 0.0985 | 0.1307 | 0.1626 | 0.1942 | 0.2411 | 0.3182 | 0.4690 | 0.7598 | 1.0400 | 1.4456 |
| 0.85 | 0.9946 | 0.9725 | 0.9436 | 0.8810 | 0.0661 | 0.0983 | 0.1301 | 0.1614 | 0.1924 | 0.2382 | 0.3132 | 0.4591 | 0.7388 | 1.0071 | 1.3943 |
| 0.90 | 0.9954 | 0.9768 | 0.9528 | 0.9015 | 0.7800 | 0.1006 | 0.1321 | 0.1630 | 0.1935 | 0.2383 | 0.3114 | 0.4527 | 0.7220 | 0.9793 | 1.3496 |
| 0.93 | 0.9959 | 0.9790 | 0.9573 | 0.9115 | 0.8059 | 0.6635 | 0.1359 | 0.1664 | 0.1963 | 0.2405 | 0.3122 | 0.4507 | 0.7138 | 0.9648 | 1.3257 |
| 0.95 | 0.9961 | 0.9803 | 0.9600 | 0.9174 | 0.8206 | 0.6967 | 0.1410 | 0.1705 | 0.1998 | 0.2432 | 0.3138 | 0.4501 | 0.7092 | 0.9561 | 1.3108 |
| 0.97 | 0.9963 | 0.9815 | 0.9625 | 0.9227 | 0.8338 | 0.7240 | 0.5580 | 0.1779 | 0.2055 | 0.2474 | 0.3164 | 0.4504 | 0.7052 | 0.9480 | 1.2968 |
| 0.98 | 0.9965 | 0.9821 | 0.9637 | 0.9253 | 0.8398 | 0.7360 | 0.5887 | 0.1844 | 0.2097 | 0.2503 | 0.3182 | 0.4508 | 0.7035 | 0.9442 | 1.2901 |
| 0.99 | 0.9966 | 0.9826 | 0.9648 | 0.9277 | 0.8455 | 0.7471 | 0.6138 | 0.1959 | 0.2154 | 0.2538 | 0.3204 | 0.4514 | 0.7018 | 0.9406 | 1.2835 |
| 1.00 | 0.9967 | 0.9832 | 0.9659 | 0.9300 | 0.8509 | 0.7574 | 0.6353 | 0.2901 | 0.2237 | 0.2583 | 0.3229 | 0.4522 | 0.7004 | 0.9372 | 1.2772 |
| 1.01 | 0.9968 | 0.9837 | 0.9669 | 0.9322 | 0.8561 | 0.7671 | 0.6542 | 0.4648 | 0.2370 | 0.2640 | 0.3260 | 0.4533 | 0.6991 | 0.9339 | 1.2710 |
| 1.02 | 0.9969 | 0.9842 | 0.9679 | 0.9343 | 0.8610 | 0.7761 | 0.6710 | 0.5146 | 0.2629 | 0.2715 | 0.3297 | 0.4547 | 0.6980 | 0.9307 | 1.2650 |
| 1.05 | 0.9971 | 0.9855 | 0.9707 | 0.9401 | 0.8743 | 0.8002 | 0.7130 | 0.6026 | 0.4437 | 0.3131 | 0.3452 | 0.4604 | 0.6956 | 0.9222 | 1.2481 |
| 1.10 | 0.9975 | 0.9874 | 0.9747 | 0.9485 | 0.8930 | 0.8323 | 0.7649 | 0.6880 | 0.5984 | 0.4580 | 0.3953 | 0.4770 | 0.6950 | 0.9110 | 1.2232 |
| 1.15 | 0.9978 | 0.9891 | 0.9780 | 0.9554 | 0.9081 | 0.8576 | 0.8032 | 0.7443 | 0.6803 | 0.5798 | 0.4760 | 0.5042 | 0.6987 | 0.9033 | 1.2021 |
| 1.20 | 0.9981 | 0.9904 | 0.9808 | 0.9611 | 0.9205 | 0.8779 | 0.8330 | 0.7858 | 0.7363 | 0.6605 | 0.5605 | 0.5425 | 0.7069 | 0.8990 | 1.1844 |
| 1.30 | 0.9985 | 0.9926 | 0.9852 | 0.9702 | 0.9396 | 0.9083 | 0.8764 | 0.8438 | 0.8111 | 0.7624 | 0.6908 | 0.6344 | 0.7358 | 0.8998 | 1.1580 |
| 1.40 | 0.9988 | 0.9942 | 0.9884 | 0.9768 | 0.9534 | 0.9298 | 0.9062 | 0.8827 | 0.8595 | 0.8256 | 0.7753 | 0.7202 | 0.7761 | 0.9112 | 1.1419 |
| 1.50 | 0.9991 | 0.9954 | 0.9909 | 0.9818 | 0.9636 | 0.9456 | 0.9278 | 0.9103 | 0.8933 | 0.8689 | 0.8328 | 0.7887 | 0.8200 | 0.9297 | 1.1339 |
| 1.60 | 0.9993 | 0.9964 | 0.9928 | 0.9856 | 0.9714 | 0.9575 | 0.9439 | 0.9308 | 0.9180 | 0.9000 | 0.8738 | 0.8410 | 0.8617 | 0.9518 | 1.1320 |
| 1.70 | 0.9994 | 0.9971 | 0.9943 | 0.9886 | 0.9775 | 0.9667 | 0.9563 | 0.9463 | 0.9367 | 0.9234 | 0.9043 | 0.8809 | 0.8984 | 0.9745 | 1.1343 |
| 1.80 | 0.9995 | 0.9977 | 0.9955 | 0.9910 | 0.9823 | 0.9739 | 0.9659 | 0.9583 | 0.9511 | 0.9413 | 0.9275 | 0.9118 | 0.9297 | 0.9961 | 1.1391 |
| 1.90 | 0.9996 | 0.9982 | 0.9964 | 0.9929 | 0.9861 | 0.9796 | 0.9735 | 0.9678 | 0.9624 | 0.9552 | 0.9456 | 0.9359 | 0.9557 | 1.0157 | 1.1452 |
| 2.00 | 0.9997 | 0.9986 | 0.9972 | 0.9944 | 0.9892 | 0.9842 | 0.9796 | 0.9754 | 0.9715 | 0.9664 | 0.9599 | 0.9550 | 0.9772 | 1.0328 | 1.1516 |
| 2.20 | 0.9998 | 0.9992 | 0.9983 | 0.9967 | 0.9937 | 0.9910 | 0.9886 | 0.9865 | 0.9847 | 0.9826 | 0.9806 | 0.9827 | 1.0094 | 1.0600 | 1.1635 |
| 2.40 | 0.9999 | 0.9996 | 0.9991 | 0.9983 | 0.9969 | 0.9957 | 0.9948 | 0.9941 | 0.9936 | 0.9935 | 0.9945 | 1.0011 | 1.0313 | 1.0793 | 1.1728 |
| 2.60 | 1.0000 | 0.9998 | 0.9997 | 0.9994 | 0.9991 | 0.9990 | 0.9990 | 0.9993 | 0.9998 | 1.0010 | 1.0040 | 1.0137 | 1.0463 | 1.0926 | 1.1792 |
| 2.80 | 1.0000 | 1.0000 | 1.0001 | 1.0002 | 1.0007 | 1.0013 | 1.0021 | 1.0031 | 1.0042 | 1.0063 | 1.0106 | 1.0223 | 1.0565 | 1.1016 | 1.1830 |
| 3.00 | 1.0000 | 1.0002 | 1.0004 | 1.0008 | 1.0018 | 1.0030 | 1.0043 | 1.0057 | 1.0074 | 1.0101 | 1.0153 | 1.0284 | 1.0635 | 1.1075 | 1.1848 |
| 3.50 | 1.0001 | 1.0004 | 1.0008 | 1.0017 | 1.0035 | 1.0055 | 1.0075 | 1.0097 | 1.0120 | 1.0156 | 1.0221 | 1.0368 | 1.0723 | 1.1138 | 1.1834 |
| 4.00 | 1.0001 | 1.0005 | 1.0010 | 1.0021 | 1.0043 | 1.0066 | 1.0090 | 1.0115 | 1.0140 | 1.0179 | 1.0249 | 1.0401 | 1.0747 | 1.1136 | 1.1773 |

lished [e.g., 63, 123]. One reason for the equation's popularity is that the coefficients $B, C, \ldots$ can be related to parameters characterizing the intermolecular potential function. Little information is available for the third and higher virial coefficients. Two methods have, however, appeared recently for predicting $C$, the third virial coefficient [15, 77]. More often the virial equation is t...

cient. The virial equation may also be written as a power series in either $V$ or $P$, so that truncation leads to two forms. These are

$$Z = 1 + \frac{BP}{RT} \qquad (3\text{-}5.2a)$$

| No | Formula | Name | Eq. | VP A | VP B | VP C | VP D | Tmin | Tmax | LDEN | TDEN |
|----|---------|------|-----|------|------|------|------|------|------|------|------|
| 51 | I4S1 | silicon tetraiodide | | | | | | | | | |
| 52 | I4Ti | titanium tetraiodide | | | | | | | | | |
| 53 | Kr | krypton | 2 | 24.097 | 1408.77 | -2.579 | 336. | 115 | TC | 2.42 | 120 |
| 54 | NO | nitric oxide | 2 | 54.894 | 2465.78 | -7.211 | 209. | 115 | TC | 1.28 | 121 |
| 55 | NO2 | nitrogen dioxide | 2 | 55.242 | 6073.34 | -6.094 | 780. | 270 | TC | 1.45 | 293 |
| 56 | N2 | nitrogen | 1 | -6.09676 | 1.13670 | -1.04072 | -1.93306 | 63 | TC | 0.804 | 78 |
| 57 | N2O | nitrous oxide | 2 | 39.824 | 2867.98 | -4.655 | 557. | 190 | TC | 1.226 | 184 |
| 58 | Ne | neon | 1 | -6.07686 | 1.59402 | -1.06092 | 4.06656 | 25 | TC | 1.204 | 27 |
| 59 | O2 | oxygen | 1 | -6.28275 | 1.73619 | =1.81349 | -2.53645E-2 | 54 | TC | 1.149 | 90 |
| 60 | O2S | sulfur dioxide | 2 | 48.882 | 4552.50 | -5.666 | 990. | 235 | TC | 1.455 | 263 |
| 61 | O3 | ozone | 3 | 9.1225 | 1272.18 | -22.16 | | 109 | 174 | 1.356 | 161 |
| 62 | O3S | sulfur trioxide | 2 | 132.94 | 10420.1 | -17.38 | 1200. | 300 | TC | 1.78 | 318 |
| 63 | P | phosphorous | | | | | | | | | |
| 64 | Rn | radon | | | | | | | | 4.4 | 211 |
| 65 | S | sulfur | | | | | | | | | |
| 66 | Se | selenium | | | | | | | | | |
| 67 | T2 | tritium | | | | | | | | | |
| 68 | Xe | xenon | 2 | 24.809 | 1951.76 | -2.544 | 603. | 170 | TC | 3.06 | 165 |
| 69 | HBr | hydrogen bromide | 2 | 21.482 | 2394.35 | -1.843 | 653. | 200 | TC | 2.16 | 216 |
| 70 | HCl | hydrogen chloride | 2 | 31.994 | 2626.67 | -3.443 | 538. | 180 | TC | 1.193 | 188 |
| 71 | HD | hydrogen deuteride | | | | | | | | | |
| 72 | HF | hydrogen fluoride | 1 | -9.74369 | 4.68946 | -2.98358 | 9.65825 | 273 | TC | 0.967 | 293 |
| 73 | HI | hydrogen iodide | 2 | 27.264 | 3013.08 | -2.673 | 923. | 235 | TC | 2.80 | 237 |
| 74 | HF2N | difluoroamine | | | | | | | | | |
| 75 | H2 | hydrogen (equilibrium) | 1 | -5.57929 | 2.60012 | -0.85506 | 1.70503 | 14 | TC | 0.071 | 20 |

| No | Formula | Name | MolWt | Tfp K | Tb K | Tc K | Pc bar | Vc cm³/mol | Zc | Omega | Dipm debye |
|----|---------|------|-------|-------|------|------|--------|------------|----|-------|------------|
| 76 | H2 | hydrogen (normal) | 2.016 | 14.0 | 20.4 | 33.2 | 13.0 | 65.1 | 0.306 | -0.218 | 0.0 |
| 77 | H2O | water | 18.015 | 273.2 | 373.2 | 647.3 | 221.2 | 57.1 | 0.235 | 0.344 | 1.8 |
| 78 | H2S | hydrogen sulfide | 34.080 | 189.6 | 213.5 | 373.2 | 89.4 | 98.6 | 0.284 | 0.081 | 0.9 |
| 79 | H3As | arsine | 77.946 | 159.7 | 218. | 373.1 | | | | | 0.2 |
| 80 | H3N | ammonia | 17.031 | 195.4 | 239.8 | 405.5 | 113.5 | 72.5 | 0.244 | 0.250 | 1.5 |
| 81 | H3P | phosphine | 33.998 | 140. | 185.4 | 324.5 | 65.4 | | | 0.038 | 0.6 |
| 82 | H4ClN | ammonium chloride | 53.492 | | 793. | 882. | 16.4 | | | 3.92 | |
| 83 | H4ClP | phosphonium chloride | 70.459 | | 246. | 322.3 | 73.7 | | | 1.64 | |
| 84 | H4N2 | hydrazine | 32.045 | 274.7 | 386.7 | 653. | 147. | | | 0.316 | 3.0 |
| 85 | H4Si | silane | 32.122 | 88.2 | 161. | 269.7 | 48.4 | | | 0.068 | 0.0 |
| 86 | H6B2 | diborane | 27.668 | 108. | 185.6 | 289.8 | 40.5 | | | 0.217 | 0.0 |
| 87 | CBrClF2 | bromochlorodifluoromethane | 165.364 | | 269. | 426.9 | 42.5 | 245.5 | 0.294 | 0.184 | |
| 88 | CBr2F2 | dibromodifluoromethane | 209.815 | 132. | 298. | 471.3 | 41.3 | | | | 0.7 |
| 89 | CBrF3 | trifluorobromomethane | 148.910 | | 215.3 | 340.2 | 39.7 | 195.9 | 0.275 | 0.171 | 0.7 |
| 90 | CClF3 | chlorotrifluoromethane | 104.459 | 92.0 | 193.2 | 302.0 | 38.7 | 180.4 | 0.278 | 0.198 | 0.5 |
| 91 | CCl2F2 | dichlorodifluoromethane | 120.914 | 115.4 | 245.2 | 385.0 | 41.4 | 216.7 | 0.280 | 0.204 | 0.5 |
| 92 | CCl2O | phosgene | 98.916 | 145.0 | 281. | 455. | 56.7 | 190.1 | 0.285 | 0.205 | 1.1 |
| 93 | CCl3F | trichlorofluoromethane | 137.368 | 162.0 | 296.9 | 471.2 | 44.1 | 247.8 | 0.279 | 0.189 | 0.5 |
| 94 | CCl4 | carbon tetrachloride | 153.823 | 250. | 349.9 | 556.4 | 45.6 | 275.9 | 0.272 | 0.193 | 0.0 |
| 95 | CD4 | deuteromethane | 20.071 | | 111.7 | 189.2 | 46.6 | 98.2 | 0.291 | 0.032 | 0.0 |
| 96 | CF4 | carbon tetrafluoride | 88.005 | 86.4 | 145.1 | 227.6 | 37.4 | 139.6 | 0.276 | 0.177 | 0.0 |
| 97 | CO | carbon monoxide | 28.010 | 68.1 | 81.7 | 132.9 | 35.0 | 93.2 | 0.295 | 0.066 | 0.1 |
| 98 | COS | carbonyl sulfide | 60.070 | 134.3 | 223. | 378.8 | 63.5 | 136.3 | 0.275 | 0.105 | 0.7 |
| 99 | CO2 | carbon dioxide | 44.010 | 216.6 | | 304.1 | 73.8 | 93.9 | 0.274 | 0.239 | 0.0 |
| 100 | CS2 | carbon disulfide | 76.131 | 161.3 | 319. | 552. | 79.0 | 160. | 0.276 | 0.109 | 0.0 |

TABLE 3-3   Values of $Z^{(1)}$

| $T_r$ | 0.010 | 0.050 | 0.100 | 0.200 | 0.400 | 0.600 | 0.800 | 1.000 | 1.200 | 1.500 | 2.000 | 3.000 | 5.000 | 7.000 | 10.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.30 | -0.0008 | -0.0040 | -0.0081 | -0.0161 | -0.0323 | -0.0484 | -0.0645 | -0.0806 | -0.0966 | -0.1207 | -0.1608 | -0.2407 | -0.3996 | -0.5572 | -0.7915 |
| 0.35 | -0.0009 | -0.0046 | -0.0093 | -0.0185 | -0.0370 | -0.0554 | -0.0738 | -0.0921 | -0.1105 | -0.1379 | -0.1834 | -0.2738 | -0.4523 | -0.6279 | -0.8863 |
| 0.40 | -0.0010 | -0.0048 | -0.0095 | -0.0190 | -0.0380 | -0.0570 | -0.0758 | -0.0946 | -0.1134 | -0.1414 | -0.1879 | -0.2799 | -0.4603 | -0.6365 | -0.8936 |
| 0.45 | -0.0009 | -0.0047 | -0.0094 | -0.0187 | -0.0374 | -0.0560 | -0.0745 | -0.0929 | -0.1113 | -0.1387 | -0.1840 | -0.2734 | -0.4475 | -0.6162 | -0.8606 |
| 0.50 | -0.0009 | -0.0045 | -0.0090 | -0.0181 | -0.0360 | -0.0539 | -0.0716 | -0.0893 | -0.1069 | -0.1330 | -0.1762 | -0.2611 | -0.4253 | -0.5831 | -0.8099 |
| 0.55 | -0.0314 | -0.0043 | -0.0086 | -0.0172 | -0.0343 | -0.0513 | -0.0682 | -0.0849 | -0.1015 | -0.1263 | -0.1669 | -0.2465 | -0.3991 | -0.5446 | -0.7521 |
| 0.60 | -0.0205 | -0.0041 | -0.0082 | -0.0164 | -0.0326 | -0.0487 | -0.0646 | -0.0803 | -0.0960 | -0.1192 | -0.1572 | -0.2312 | -0.3718 | -0.5047 | -0.6928 |
| 0.65 | -0.0137 | -0.0772 | -0.0078 | -0.0156 | -0.0309 | -0.0461 | -0.0611 | -0.0759 | -0.0906 | -0.1122 | -0.1476 | -0.2160 | -0.3447 | -0.4653 | -0.6346 |
| 0.70 | -0.0093 | -0.0507 | -0.1161 | -0.0148 | -0.0294 | -0.0438 | -0.0579 | -0.0718 | -0.0855 | -0.1057 | -0.1385 | -0.2013 | -0.3184 | -0.4270 | -0.5785 |
| 0.75 | -0.0064 | -0.0339 | -0.0744 | -0.0143 | -0.0282 | -0.0417 | -0.0550 | -0.0681 | -0.0808 | -0.0996 | -0.1298 | -0.1872 | -0.2929 | -0.3901 | -0.5250 |
| 0.80 | -0.0044 | -0.0228 | -0.0487 | -0.1160 | -0.0272 | -0.0401 | -0.0526 | -0.0648 | -0.0767 | -0.0940 | -0.1217 | -0.1736 | -0.2682 | -0.3545 | -0.4740 |
| 0.85 | -0.0029 | -0.0152 | -0.0319 | -0.0715 | -0.0268 | -0.0391 | -0.0509 | -0.0622 | -0.0731 | -0.0888 | -0.1138 | -0.1602 | -0.2439 | -0.3201 | -0.4254 |
| 0.90 | -0.0019 | -0.0099 | -0.0205 | -0.0442 | -0.1118 | -0.0396 | -0.0503 | -0.0604 | -0.0701 | -0.0840 | -0.1059 | -0.1463 | -0.2195 | -0.2862 | -0.3788 |
| 0.93 | -0.0015 | -0.0075 | -0.0154 | -0.0326 | -0.0763 | -0.1662 | -0.0514 | -0.0602 | -0.0687 | -0.0810 | -0.1007 | -0.1374 | -0.2045 | -0.2661 | -0.3516 |
| 0.95 | -0.0012 | -0.0062 | -0.0126 | -0.0262 | -0.0589 | -0.1110 | -0.0540 | -0.0607 | -0.0678 | -0.0788 | -0.0967 | -0.1310 | -0.1943 | -0.2526 | -0.3339 |
| 0.97 | -0.0010 | -0.0050 | -0.0101 | -0.0208 | -0.0450 | -0.0770 | -0.1647 | -0.0623 | -0.0669 | -0.0759 | -0.0921 | -0.1240 | -0.1837 | -0.2391 | -0.3163 |
| 0.98 | -0.0009 | -0.0044 | -0.0090 | -0.0184 | -0.0390 | -0.0641 | -0.1100 | -0.0641 | -0.0661 | -0.0740 | -0.0893 | -0.1202 | -0.1783 | -0.2322 | -0.3075 |
| 0.99 | -0.0008 | -0.0039 | -0.0079 | -0.0161 | -0.0335 | -0.0531 | -0.0796 | -0.0680 | -0.0646 | -0.0715 | -0.0861 | -0.1162 | -0.1728 | -0.2254 | -0.2989 |
| 1.00 | -0.0007 | -0.0034 | -0.0069 | -0.0140 | -0.0285 | -0.0435 | -0.0588 | -0.0879 | -0.0609 | -0.0678 | -0.0824 | -0.1118 | -0.1672 | -0.2185 | -0.2902 |
| 1.01 | -0.0006 | -0.0030 | -0.0060 | -0.0120 | -0.0240 | -0.0351 | -0.0429 | -0.0223 | -0.0473 | -0.0621 | -0.0778 | -0.1072 | -0.1615 | -0.2116 | -0.2816 |
| 1.02 | -0.0005 | -0.0026 | -0.0051 | -0.0102 | -0.0198 | -0.0277 | -0.0303 | -0.0062 | 0.0227 | -0.0524 | -0.0722 | -0.1021 | -0.1556 | -0.2047 | -0.2731 |
| 1.05 | -0.0003 | -0.0015 | -0.0029 | -0.0054 | -0.0092 | -0.0097 | -0.0032 | 0.0220 | 0.1059 | 0.0451 | -0.0432 | -0.0838 | -0.1370 | -0.1835 | -0.2476 |
| 1.10 | -0.0000 | -0.0000 | 0.0001 | 0.0007 | 0.0038 | 0.0106 | 0.0236 | 0.0476 | 0.0897 | 0.1630 | 0.0698 | -0.0373 | -0.1021 | -0.1469 | -0.2056 |
| 1.15 | 0.0002 | 0.0011 | 0.0023 | 0.0052 | 0.0127 | 0.0237 | 0.0396 | 0.0625 | 0.0943 | 0.1548 | 0.1667 | 0.0332 | -0.0611 | -0.1084 | -0.1642 |
| 1.20 | 0.0004 | 0.0019 | 0.0039 | 0.0084 | 0.0190 | 0.0326 | 0.0499 | 0.0719 | 0.0991 | 0.1477 | 0.1990 | 0.1095 | -0.0141 | -0.0678 | -0.1231 |
| 1.30 | 0.0006 | 0.0030 | 0.0061 | 0.0125 | 0.0267 | 0.0429 | 0.0612 | 0.0819 | 0.1048 | 0.1420 | 0.1991 | 0.2079 | 0.0875 | 0.0176 | -0.0423 |
| 1.40 | 0.0007 | 0.0036 | 0.0072 | 0.0147 | 0.0306 | 0.0477 | 0.0661 | 0.0857 | 0.1063 | 0.1383 | 0.1894 | 0.2397 | 0.1737 | 0.1008 | 0.0350 |
| 1.50 | 0.0008 | 0.0039 | 0.0078 | 0.0158 | 0.0323 | 0.0497 | 0.0677 | 0.0864 | 0.1055 | 0.1345 | 0.1806 | 0.2433 | 0.2309 | 0.1717 | 0.1058 |
| 1.60 | 0.0008 | 0.0040 | 0.0080 | 0.0162 | 0.0330 | 0.0501 | 0.0677 | 0.0855 | 0.1035 | 0.1303 | 0.1729 | 0.2381 | 0.2631 | 0.2255 | 0.1673 |
| 1.70 | 0.0008 | 0.0040 | 0.0081 | 0.0163 | 0.0329 | 0.0497 | 0.0667 | 0.0836 | 0.1008 | 0.1259 | 0.1658 | 0.2305 | 0.2788 | 0.2628 | 0.2179 |
| 1.80 | 0.0008 | 0.0040 | 0.0081 | 0.0162 | 0.0325 | 0.0488 | 0.0652 | 0.0816 | 0.0978 | 0.1216 | 0.1593 | 0.2224 | 0.2846 | 0.2871 | 0.2576 |
| 1.90 | 0.0008 | 0.0040 | 0.0079 | 0.0159 | 0.0318 | 0.0477 | 0.0635 | 0.0792 | 0.0947 | 0.1173 | 0.1532 | 0.2144 | 0.2848 | 0.3017 | 0.2876 |
| 2.00 | 0.0008 | 0.0039 | 0.0078 | 0.0155 | 0.0310 | 0.0464 | 0.0617 | 0.0767 | 0.0916 | 0.1133 | 0.1476 | 0.2069 | 0.2819 | 0.3097 | 0.3096 |
| 2.20 | 0.0007 | 0.0037 | 0.0074 | 0.0147 | 0.0293 | 0.0437 | 0.0579 | 0.0719 | 0.0857 | 0.1057 | 0.1374 | 0.1932 | 0.2720 | 0.3135 | 0.3355 |
| 2.40 | 0.0007 | 0.0035 | 0.0070 | 0.0139 | 0.0276 | 0.0411 | 0.0544 | 0.0675 | 0.0803 | 0.0989 | 0.1285 | 0.1812 | 0.2602 | 0.3089 | 0.3459 |
| 2.60 | 0.0007 | 0.0033 | 0.0066 | 0.0131 | 0.0260 | 0.0387 | 0.0512 | 0.0634 | 0.0754 | 0.0929 | 0.1207 | 0.1706 | 0.2484 | 0.3009 | 0.3475 |
| 2.80 | 0.0006 | 0.0031 | 0.0062 | 0.0124 | 0.0245 | 0.0365 | 0.0483 | 0.0598 | 0.0711 | 0.0876 | 0.1138 | 0.1613 | 0.2372 | 0.2915 | 0.3443 |
| 3.00 | 0.0006 | 0.0029 | 0.0059 | 0.0117 | 0.0232 | 0.0345 | 0.0456 | 0.0565 | 0.0672 | 0.0828 | 0.1076 | 0.1529 | 0.2268 | 0.2817 | 0.3385 |
| 3.50 | 0.0005 | 0.0026 | 0.0052 | 0.0103 | 0.0204 | 0.0303 | 0.0401 | 0.0497 | 0.0591 | 0.0728 | 0.0949 | 0.1356 | 0.2042 | 0.2584 | 0.3194 |
| 4.00 | 0.0005 | 0.0023 | 0.0046 | 0.0091 | 0.0182 | 0.0270 | 0.0357 | 0.0443 | 0.0527 | 0.0651 | 0.0849 | 0.1219 | 0.1857 | 0.2378 | 0.2994 |

For the evaluation of $B$ from experimental data, however, Eq. (3-5.1) should be used, i.e.,

$$B = \lim_{1/V \to 0} \left(\frac{\partial Z}{\partial 1/V}\right)_T \qquad (3\text{-}5.3)$$

As the density approaches zero, Eqs. (3-5.2a) and (3-5.2b) become identical, and at low densities, both forms of Eq. (3-5.2) approximate true behavior. But neither Eq. (3-5.2a) nor (3-5.2b) should be used if $\rho > \rho_c/2$ or if $V_{r_i}$ as defined in Figs. 3-1 to 3-3 is less than about 0.5. When $\rho = \rho_c/2$, Eq. (3-5.2a) predicts a Z which is too high and Eq. (3-5.2b) predicts

| No | Formula | Name | CPVAP A | CPVAP B | CPVAP C | CPVAP D | DELHF | DELG |
|----|---------|------|---------|---------|---------|---------|-------|------|
| 76 | H2 | hydrogen (normal) | | | | | | |
| 77 | H2O | water | 3.224E+1 | 1.924E-3 | 1.055E-5 | -3.596E-9 | -2.420E+5 | -2.288E+5 |
| 78 | H2S | hydrogen sulfide | 3.194E+1 | 1.436E-3 | 2.432E-5 | -1.176E-8 | -2.018E+4 | -3.308E+4 |
| 79 | H3As | arsine | | | | | 1.825E+5 | 1.578E+5 |
| 80 | H3N | ammonia | 2.731E+1 | 2.383E-2 | 1.707E-5 | -1.185E-8 | -4.572E+4 | -1.616E+4 |
| 81 | H3P | phosphine | 2.323E+1 | 4.401E-2 | 1.303E-5 | -1.593E-8 | 2.290E+4 | 2.541E+4 |
| 82 | H4ClN | ammonium chloride | | | | | | |
| 83 | H4ClP | phosphonium chloride | | | | | | |
| 84 | H4N2 | hydrazine | 9.768E+0 | 1.895E-1 | -1.657E-4 | 6.025E-8 | 9.525E+4 | 1.586E+5 |
| 85 | H4Si | silane | 1.118E+1 | 1.220E-1 | -5.548E-5 | 6.840E-9 | 3.266E+4 | 5.518E+4 |
| 86 | H6B2 | diborane | | | | | 3.140E+4 | 8.332E+4 |
| 87 | CBrClF2 | bromochlorodifluoromethane | | | | | | |
| 88 | CBr2F2 | dibromodifluoromethane | | | | | | |
| 89 | CBrF3 | trifluorobromomethane | 2.188E+1 | 2.159E-1 | -2.114E-4 | 7.464E-8 | -6.494E+5 | -6.975E+5 |
| 90 | CClF3 | chlorotrifluoromethane | 2.281E+1 | 1.911E-1 | -1.576E-4 | 4.459E-8 | -6.950E+5 | -6.544E+5 |
| 91 | CCl2F2 | dichlorodifluoromethane | 3.160E+1 | 1.782E-1 | -1.509E-4 | 4.342E-8 | -4.815E+5 | -4.425E+5 |
| 92 | CCl2O | phosgene | 2.809E+1 | 1.361E-1 | -1.374E-4 | 5.070E-8 | -2.211E+5 | -2.069E+5 |
| 93 | CCl3F | trichlorofluoromethane | 4.098E+1 | 1.668E-1 | -1.410E-4 | 4.146E-8 | -2.847E+5 | -2.455E+5 |
| 94 | CCl4 | carbon tetrachloride | 4.072E+1 | 2.049E-1 | -2.270E-4 | 8.843E-8 | -1.005E+5 | -5.828E+4 |
| 95 | CD4 | deuteromethane | 1.249E+1 | 1.010E-1 | -2.199E-5 | -8.458E-9 | -8.830E+4 | -5.954E+4 |
| 96 | CF4 | carbon tetrafluoride | 1.398E+1 | 2.026E-1 | -1.625E-4 | 4.513E-8 | -9.337E+5 | -8.890E+5 |
| 97 | CO | carbon monoxide | 3.087E+1 | -1.285E-2 | 2.789E-5 | -1.272E-8 | -1.106E+5 | -1.374E+5 |
| 98 | COS | carbonyl sulfide | 2.357E+1 | 7.984E-2 | -7.017E-5 | 2.453E-8 | -1.385E+5 | -1.658E+5 |
| 99 | CO2 | carbon dioxide | 1.980E+1 | 7.344E-2 | -5.602E-5 | 1.715E-8 | -3.938E+5 | -3.946E+5 |
| 100 | CS2 | carbon disulfide | 2.744E+1 | 8.127E-2 | -7.666E-5 | 2.673E-8 | 1.171E+5 | 6.695E+4 |

| No | Formula | Name | Eq. | VP A | VP B | VP C | VP D | Tmin | Tmax | LDEN | TDEN |
|----|---------|------|-----|------|------|------|------|------|------|------|------|
| 76 | H2 | hydrogen (normal) | | | | | | | | | |
| 77 | H2O | water | 1 | -7.76451 | 1.45838 | -2.77580 | -1.23303 | 275 | TC | 0.998 | 293 |
| 78 | H2S | hydrogen sulfide | 2 | 36.067 | 3132.31 | -3.985 | 653. | 205 | TC | 0.993 | 214 |
| 79 | H3As | arsine | | | | | | | | 1.604 | 209 |
| 80 | H3N | ammonia | 2 | 45.327 | 4104.67 | -5.146 | 615. | 220 | TC | 0.639 | 273 |
| 81 | H3P | phosphine | | | | | | | | 1.529 | 298 |
| 82 | H4ClN | ammonium chloride | | | | | | | | | |
| 83 | H4ClP | phosphonium chloride | | | | | | | | | |
| 84 | H4N2 | hydrazine | 2 | 49.476 | 6951.84 | -5.286 | 1222. | 350 | TC | 1.008 | 293 |
| 85 | H4Si | silane | | | | | | | | 0.68 | 88 |
| 86 | H6B2 | diborane | 3 | 8.0390 | 1200.78 | -31.22 | | 118 | 181 | 0.470 | 153 |
| 87 | CBrClF2 | bromochlorodifluoromethane | 3 | 9.1295 | 2154.39 | -32.87 | | 178 | 283 | | |
| 88 | CBr2F2 | dibromodifluoromethane | 3 | 9.8485 | 2720.78 | -19.35 | | 247 | 296 | 2.462 | 288 |
| 89 | CBrF3 | trifluorobromomethane | | | | | | | | 1.538 | 298 |
| 90 | CClF3 | chlorotrifluoromethane | 1 | -6.78845 | 1.24435 | -2.32601 | 1.45543 | 233 | TC | 1.298 | 243 |
| 91 | CCl2F2 | dichlorodifluoromethane | 1 | -7.01657 | 1.73224 | -2.97909 | -0.37723 | 155 | TC | 1.750 | 158 |
| 92 | CCl2O | phosgene | 1 | -7.08177 | 1.60461 | -2.57153 | -1.88377 | 216 | TC | 1.381 | 293 |
| 93 | CCl3F | trichlorofluoromethane | 2 | 42.089 | 4464.14 | -4.753 | 2138. | 260 | TC | 1.494 | 290 |
| 94 | CCl4 | carbon tetrachloride | 1 | -7.07139 | 1.71497 | -2.89930 | -2.49466 | 250 | TC | 1.584 | 298 |
| 95 | CD4 | deuteromethane | | | | | | | | | |
| 96 | CF4 | carbon tetrafluoride | 3 | 9.4341 | 1244.55 | -13.06 | | 93 | 148 | 1.33 | 193 |
| 97 | CO | carbon monoxide | 1 | -6.20798 | 1.27885 | -1.34533 | -2.56842 | 71 | TC | 0.803 | 81 |
| 98 | COS | carbonyl sulfide | 1 | -6.40952 | 1.21015 | -1.54976 | -2.10074 | 162 | TC | 1.274 | 174 |
| 99 | CO2 | carbon dioxide | 1 | -6.95626 | 1.19695 | -3.12614 | 2.99448 | 217 | TC | | |
| 100 | CS2 | carbon disulfide | 1 | -6.63896 | 1.20395 | -0.37653 | -4.32820 | 277 | TC | 1.293 | 273 |

Approximate equations showing the relationship between the friction factor and the Reynolds number in the turbulent-flow region have been developed. Two of these equations follow:

For smooth pipe or tubes,

$$f = \frac{0.046}{(N_{Re})^{0.2}} \qquad (7)$$

For new iron or steel pipe,

$$f = \frac{0.04}{(N_{Re})^{0.16}} \qquad (8)$$

**INTEGRATED FORM OF THE FANNING EQUATION.** If the linear velocity, density, and viscosity of the flowing fluid remain constant and the pipe diameter is uniform over a total pipe length $L$, Eq. (5) can be integrated to give the following result:

$$F = \frac{-\Delta p_f}{\rho} = \frac{2fV^2 L}{g_c D} \qquad (9)$$

In a strict sense, Eq. (9) is limited to conditions in which the flowing fluid is noncompressible and the temperature of the fluid is constant. When dealing with compressible fluids, such as air, steam, or any gas, it is good engineering practice to use Eq. (9) only if the pressure drop over the system is less than 10 percent of the initial pressure. If a change in the fluid temperature occurs, Eq. (9) should not be used in the form indicated unless the total change in the fluid viscosity is less than approximately 50 percent based on the maximum viscosity.†‡ If Eq. (9) is used when pressure changes or temperature changes are involved, the best accuracy is obtained by using the linear velocity, density, and viscosity of the fluid as determined at the average temperature and pressure. Exact results for compressible fluids or nonisothermal flow can be obtained from the Fanning equation by integrating the differential expression, taking all changes into consideration.

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†Overall effects of temperature on the friction factor are more important in the streamline-flow range where $f$ is directly proportional to the viscosity than in the turbulent-flow range where $f$ is approximately proportional to $\mu^{0.16}$.

‡For heating or cooling of fluids, a temperature gradient must exist from the pipe wall across the flowing fluid. A simplified design procedure for this case is as follows: When temperature and viscosity changes must be taken into consideration, the friction factor (Fig. 14-1) should be taken as the isothermal friction factor (Fig. 14-1) based on the arithmetic-average temperature of the fluid divided by a correction factor $\phi$, where $\phi = 1.1(\mu_a/\mu_w)^{0.25}$ when $DG/\mu_a$ is less than 2100 and $\phi = 1.02(\mu_a/\mu_w)^{0.14}$ when $DG/\mu_a$ is greater than 2100. $[G = $ mass velocity, lb/(h)(ft$^2$ of cross-sectional area); $\mu_a = $ viscosity of fluid at average bulk temperature, lb/(s)(ft); $\mu_w = $ viscosity of fluid at temperature of wall, lb/(s)(ft).]
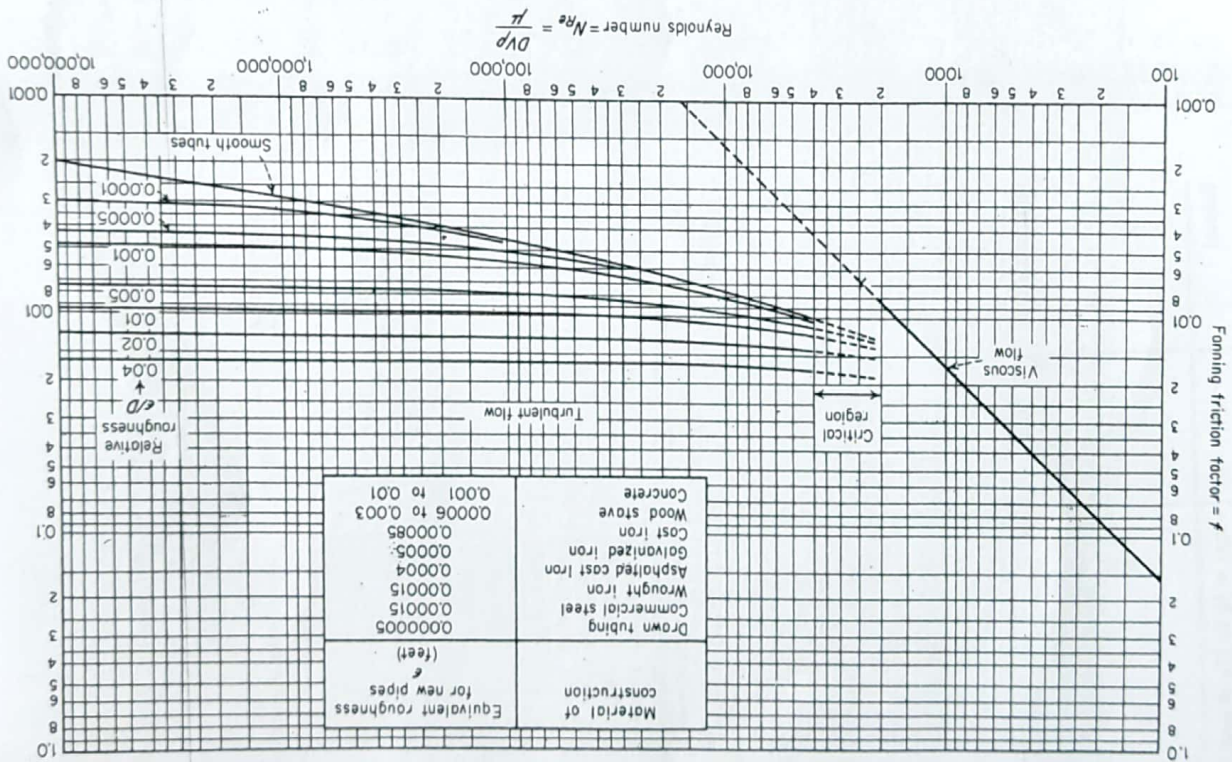
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**FIGURE 14-1**

Fanning friction factors for long straight pipes. [Based on L. F. Moody, *Trans. ASME*, **66**:671–684 (1944).]

**TABLE 1**
**Expressions for evaluating frictional losses in the flow of fluids through conduits**

For noncircular, cross-sectional area and turbulent flow, replace $D$ by $4R_H = 4$ (cross-sectional flow area/wetted perimeter).
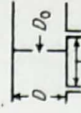
| Friction caused by | General expression for frictional loss | Limited expression and remarks |
|---|---|---|
| Flow through long straight pipe of constant cross-sectional area | $dF = \dfrac{2fV^2\,dL}{g_c D}$ | For case in which fluid is essentially noncompressible and temperature is constant $F = \dfrac{2fV^2 L}{g_c D}$ |
| Sudden enlargement | $F_e = \dfrac{(V_1 - V_2)^2}{2\alpha g_c}$ | The following values for α may be used in design calculations: turbulent flow, α = 1; streamline flow, α = 0.5 $V_1 \rightarrow V_2$ |
| Sudden contraction | $F_c = \dfrac{K_c V_2^2}{2\alpha g_c}$ | The following values for α may be used in design calculations: turbulent flow, α = 1; streamline flow, α = 0.5  $A_1\; V_2\; A_2$  For $\dfrac{A_2}{A_1} < 0.715$, $K_c = 0.4\left(1.25 - \dfrac{A_2}{A_1}\right)$  For $\dfrac{A_2}{A_1} > 0.715$, $K_c = 0.75\left(1 - \dfrac{A_2}{A_1}\right)$  For conical or rounded shape, $K_c = 0.05$ |
| Fittings, valves, etc. | $F = \dfrac{2fV^2 L_e}{g_c D}$ | $L_e/D$ per fitting (dimensionless): 45° elbows 15; 90° elbows, std. radius 32; 90° elbows, medium radius 26; 90° elbows, long sweep 20; 90° square elbows 60; 180° close-return bends 75 |

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**TABLE 1**
**Expressions for evaluating frictional losses in the flow of fluids through conduits** (*Continued*)

| Friction caused by | General expression for frictional loss | Limited expression and remarks |
|---|---|---|
| Fittings, valves, etc. | $F = \dfrac{2fV^2 L_e}{g_c D}$ | $L_e/D$ per fitting (dimensionless): 180° medium-radius return bends 50; Tee (used as elbow, entering run) 60; Tee (used as elbow, entering branch) 90; Couplings Negligible; Unions Negligible; Gate valves, open 7; Globe valves, open 300; Angle valves, open 170; Water meters, disk 400; Water meters, piston 600; Water meters, impulse wheel 300 |
| Sharp-edged orifice | $-\Delta p_f = F\rho$ | $\dfrac{\Delta p_f(100)}{\Delta p \text{ across orifice}} = \%$ ; $\dfrac{D_0}{D}$ / Δp across orifice: 0.8 → 40; 0.7 → 52; 0.6 → 63; 0.5 → 73; 0.4 → 81; 0.3 → 89; 0.2 → 95. Measured Δp across orifice |
| Rounded orifice | $F = \dfrac{(V_0 - V_2)^2}{2\alpha g_c}$ | The following values for α may be used in design calculations: turbulent flow, α = 1; streamline flow, α = 0.5 $V_0\; V_2$ |
| Venturi | $-\Delta p_f = F\rho$ | $-\Delta p_f = \tfrac{1}{8}$ to $\tfrac{1}{10}$ of total pressure drop from upstream section to venturi throat |

PRESSURE, PSIA →

Temperature °F

$K = Y/X$

$K = Y/$
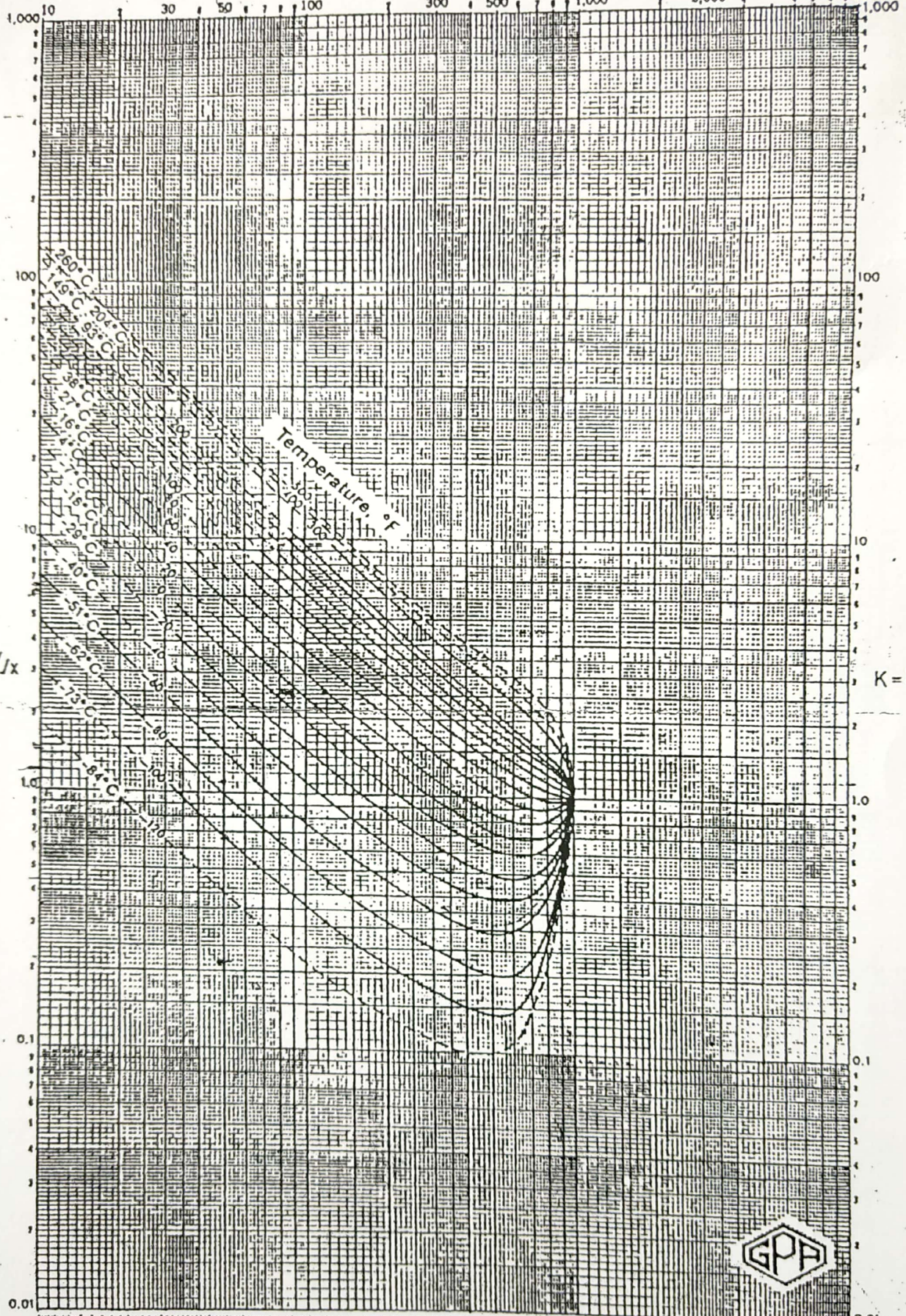
PRESSURE, kPa (abs)

SI

18 - 25

ETHANE
CONV. PRESS. 1000 PSIA

PRESSURE, PSIA
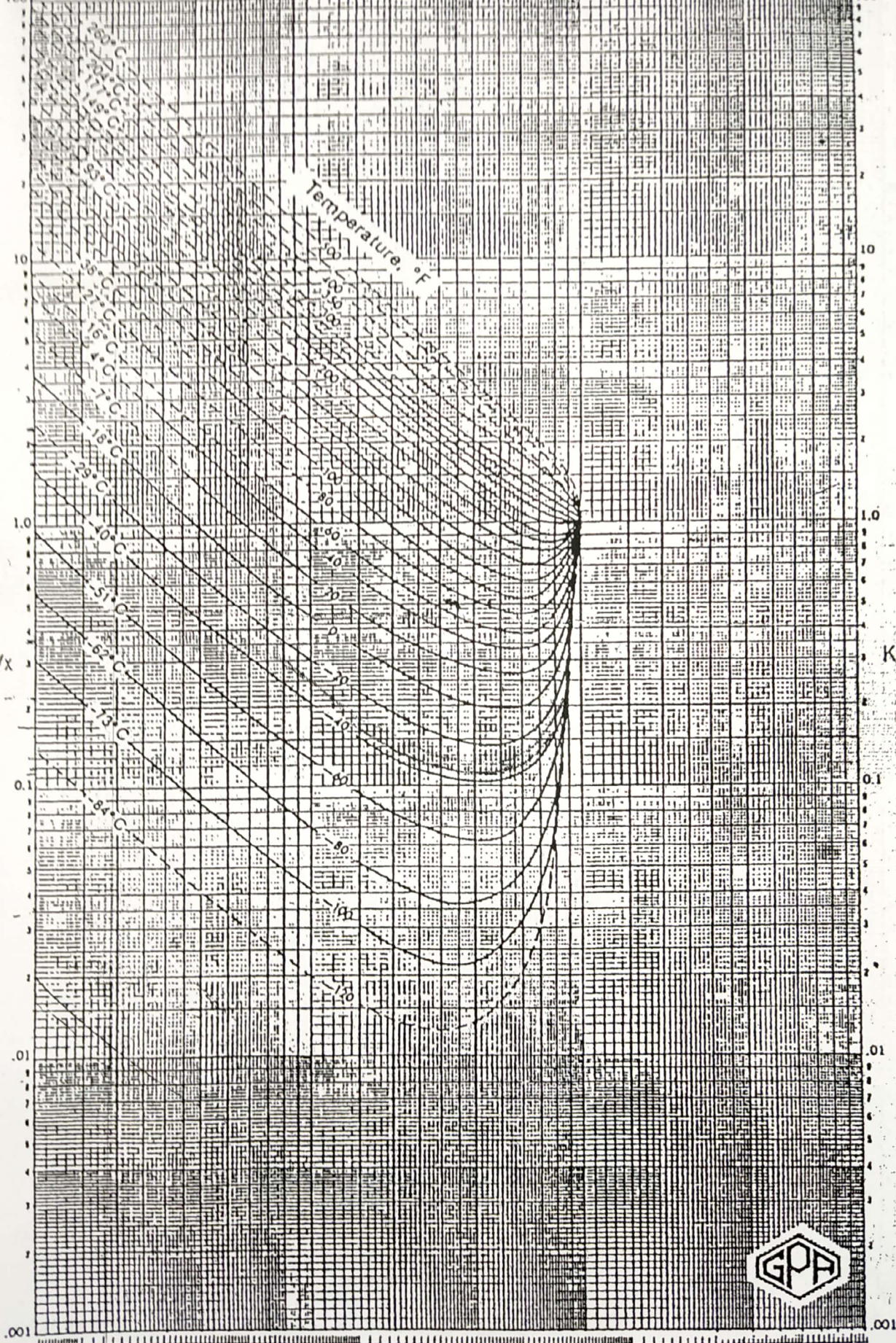
PRESSURE, kPa (abs)

SI
18 - 26

**PROPANE**
CONV. PRESS. 1000 PSIA