Department of Materials Engineering
Final Exam correction - Heat Transfer
L3/GP - January 2024
1h35

Part 1 (7pts)

1)

$$\emptyset = -\lambda S\frac{dT}{dr} = -\lambda(2\pi rl)\frac{dT}{dr}\,(\mathbf{0,5pt}) = -\,\mathrm{a}\,(1+\mathrm{b}T^2)(2\pi \mathrm{rl})\frac{dT}{dr}\,(\mathbf{0,5pt})$$

We separate the variables then we put $\emptyset = A$

$$\Rightarrow \frac{\emptyset}{2a\pi L}\frac{1}{r}dr = -\,(1+\mathrm{b}T^2)\mathrm{dT} \Rightarrow \int_{r1}^{r2}\frac{\emptyset}{2a\pi L}\frac{1}{r}dr = \int_{T1}^{T2} -(1+\mathrm{b}T^2)\mathrm{dT}\;(\mathbf{0,5pt})$$

$$\Rightarrow \frac{\emptyset}{2a\pi L}[\ln r]_{r1}^{r2} = -\left[\mathrm{T}+\frac{\mathrm{b}T^3}{3}\right]_{T1}^{T2}\;(\mathbf{0,5pt}) \Rightarrow \frac{\emptyset}{2a\pi L}\ln\frac{r2}{r1} = \left(T_1+\frac{bT_1^3}{3}\right)-\left(T_2+\frac{bT_2^3}{3}\right)\;(\mathbf{0,5pt})$$

$$\Rightarrow \frac{\emptyset}{2a\pi L}\ln\frac{r2}{r1} = (T_1-T_2)+\left(\frac{bT_1^3}{3}-\frac{bT_2^3}{3}\right) = (T_1-T_2)+\frac{b}{3}\,(T_1^3-T_2^3)$$

$$\Rightarrow \emptyset = 2a\pi L\,\frac{1}{\ln\frac{r_2}{r_1}}\left[(T_1-T_2)+\frac{b}{3}\,(T_1^3-T_2^3)\right]\;(\mathbf{0,5pt})$$

<u>2)</u>

<u>Energy equation assumptions :</u>

-Steady state$(\mathbf{0,25pt})$

-One dimensional heat conduction.

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2\mathrm{T}+\frac{q}{\lambda}$$

$$\nabla^2\mathrm{T}+\frac{q}{\lambda} = 0\;(\mathbf{0,25pt})$$

$$\frac{\delta^2 T}{\delta^2 x} = -\frac{q}{\lambda} \Rightarrow \int\frac{\delta^2 T}{\delta^2 x} = \int -\frac{q}{\lambda}\;(\mathbf{0,25pt})$$

$$\Rightarrow \frac{\delta T}{\delta x} = -\frac{q}{\lambda}\,x + B \;\Rightarrow\; T(x) = -\frac{q}{2\lambda}\,x^2 + Bx + C\;(\mathbf{0,25pt})$$

$$\Rightarrow x = 0\;, T = T_1\;\;; x = e\;, T = T_2$$

$$\Rightarrow T_1 = C\;\;, T_2 - T_1 = -\frac{q}{2\lambda}e^2 + \mathrm{Be} \Rightarrow \mathrm{B} = \frac{1}{e}(T_2-T_1)+\frac{q}{2\lambda}e\,(\mathbf{0,5pt})$$

$$\emptyset = -\lambda S\frac{dT}{dx} = -\lambda S\left(-\frac{q}{\lambda}\,x + B\right) = -\lambda S\,(-\frac{q}{\lambda}\,x+\frac{1}{e}(T_2-T_1)+\frac{q}{2\lambda}e)\;(\mathbf{0,5pt})$$

$$\boldsymbol{\phi} = -\lambda S\,(-\frac{q}{\lambda}\,x+\frac{1}{e}(T_2-T_1)+\frac{q}{2\lambda}e)$$

3) **Pi Theorem** : The parameters that manage forced convection are 7 ; Characteristic length L, Flow velocity V, Density, Dynamic viscosity, Thermal conductivity, Specific heat, Convection coefficient, with 4 Dimensions (MLTt) so effectively there are 03 dimensionless variables (7-4= 3; Reynolds, Nusselt and Prandtl- with their formulas), **(1pt)** Concerning free convection, 9 parameters ; Characteristic length, Temperature difference ΔT, Density, Dynamic viscosity, Coefficient of expansion or relaxation β,

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Acceleration of gravity, Thermal conductivity, Specific heat, Convection coefficient, with always 4 dimensions (MLTt) so there are 5 dimensionless variables (9-4=5; Reynolds, Nusselt ,Prandtl, Grashof and Rayleigh - with their formulas) **(1pt)**

Part 2 (13pts)

Problem N°1(5pts)

The passage of electric current produces heat, therefore Qp ≠0.

The heat equation in cylindrical coordinates

$$\nabla^2 T + \frac{q}{\lambda} = 0$$

$$\frac{dT^2}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q}{\lambda} = 0 \Rightarrow r\frac{dT^2}{dr^2} + \frac{dT}{dr} = -\frac{q}{\lambda}r \Rightarrow \frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{q}{\lambda}r$$

$$\Rightarrow \int \frac{d}{dr}\left(r\frac{dT}{dr}\right) = \int -\frac{q}{\lambda}r \Rightarrow \left(r\frac{dT}{dr}\right) = -\frac{q}{2\lambda}r^2 + A \Rightarrow \left(\frac{dT}{dr}\right) = -\frac{q}{2\lambda}r + \frac{A}{r}$$

$$\Rightarrow T(r) = -\frac{q}{4\lambda}r^2 + A\ln r + b \quad (1)$$

Conditions limites (r = 0, T= T_1 $\Rightarrow A = \frac{T_{1-b}}{\ln 0} = 0$

$$(1) \Rightarrow T(r) = -\frac{q}{4\lambda}r^2 + b$$

r = R & T= T_2

$$b = T_2 + \frac{q}{4\lambda}R^2$$

$$(1) \Rightarrow T(r) = -\frac{q}{4\lambda}r^2 + \frac{q}{4\lambda}R^2 + T_2$$ **(1pt)**

The temperature is maximum $\Rightarrow \frac{dT}{dr} = 0 \quad \Rightarrow \frac{dT}{dr} = -\frac{q}{2\lambda}r = 0 \Rightarrow r = 0$ **(0,5pt)**

$T_{max} \Rightarrow r=0$, $T_{max} = \frac{q}{4\lambda}R^2 + T_2$ **(0,5pt)**

$q = \frac{p}{v}$ **p=Re .I² ; v= S.L = π R² L (Re : résistance électrique (electrical resistance)** $= \frac{1}{k} \times \frac{L}{S}$
où $\frac{1}{k}$ ***c'est la résistivité éléctrique (electrical resistivity)***

$$\Rightarrow q = \left(\frac{1}{k}\right)\frac{I^2}{S^2}$$

S = π R² = (3,14)(0,85.10⁻²)² =2,268 10⁻⁴ m²

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(0, 5pt) $Tmax = (\frac{1}{k})\frac{I^2}{S^2}\frac{R^2}{4\lambda} + T_2 =$ 49,08 +90 =139,08 °C **(0,5pt)**

$$T(r) = (\frac{1}{k})\frac{I^2}{S^2}(-\frac{r^2}{4\lambda} + \frac{R^2}{4\lambda}) + T_2$$ **(1pt)**

$$I^2 = \frac{T(r) - T_2}{(-\frac{r^2}{4\lambda} + \frac{R^2}{4\lambda})} S^2 k$$

$\Rightarrow$

$$I = \sqrt{\frac{T(r)-T_2}{(-\frac{r^2}{4\lambda}+\frac{R^2}{4\lambda})} S^2 k}$$ **(0,5pt)**

T= 110°C and r = 0,3 cm.
I = 176 A**(0,5pt)**

Problem N°2 (5pts)

<u>Free Convection</u>

T_{ref} = (184+60) /2 = 122 °F =50°C **(1pt)**

P = 1,049 kg/m^3 , Cp = 1041 j/Kg.C ; λ= 0,0275(W/mC) μ= 18,85 10^{-6}(kg/m.s)

$$\Delta T = 51,11$$
$$Ra = \frac{g\beta \Delta T L^3 \rho^2 cp}{\lambda \mu}$$

$\beta = \frac{1}{T_{ref}}$ **(K)**

$$\Rightarrow Ra = \frac{9,81(3,0910^{-3})(51,11)(0,55)^3}{0,0275} \frac{(1,049)^2\ 1041}{18,85\ 10^{-6}}$$

Ra = 5,69 10^8(1pt) $10^4 \leq$ **5,69 10^8**$\leq 10^9$ $10^4 \leq Ra \leq 10^9$ **(0,5pt)**

$\overline{Nu}$ = 0,59Ra$^{0.25}$ $\Rightarrow$ **$\overline{Nu}$ = 91,12(1pt)**

$h = \frac{\overline{NU_L}}{L}.\lambda = \frac{91,12\ (0,0275)}{0,55} = 4,55$ (w/m^{2}K) **(1,5pt)**

Problem N°3 (3pts)

λ=1,8 μm ,°C = (°F-32)/1,8 T = 926,66 °C +273,15 =1199,82 K **(0,5pt)**
According to the law of Wien : λ_{max} . T = 2897,8 **(0,5pt)**

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$\lambda_{max} = \frac{2897,8}{1199,82} = 2,415\ \mu m$ **(0,5pt)**

$$q_{\lambda cn}(\lambda, T) = \frac{C_1}{\lambda^5\left[exp(C_2/\lambda T) - 1\right]}$$ **(1pt)**

$C_1 = 2\pi hc^2 = 3,742 \times 10^8\ W.\mu m^4/m^2$ $\quad C_2 = 1,439 \times 10^4\ \mu mK$

$\boldsymbol{\varphi = 2,532\ 10^4}$ **w/m²** **(0,5pt)**

Appendix problem N°2

Vertical plate		$\overline{Nu} = 0.59\ Ra^{0.25}$	$10^4 \leq Ra \leq 10^9$
		$\overline{Nu} = 0.10\ Ra^{0.33}$	$10^9 < Ra \leq 10^{13}$

(g) Nitrogen at 1 atm (101.325 kPa)

T (C)	ρ (kg/m³)	c_p (J/kg C)	k (W/mC)	μ (kg/m s)	ν (m²/s)	Pr
−50	1.523	1042	0.0201	14.12×10^{-6}	9.28×10^{-6}	0.732
0	1.253	1041	0.0239	16.57×10^{-6}	13.23×10^{-6}	0.722
50	1.049	1041	0.0275	18.85×10^{-6}	17.97×10^{-6}	0.714
100	0.897	1043	0.0309	20.96×10^{-6}	23.37×10^{-6}	0.708

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