Department of Materials Engineering
Final Exam - Heat Transfer
L3/GP - January 2026
1h30

Part 1 (5 pts)

1) Poisson equation

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{q}{\lambda} \text{ (0.25pt)}$$

Energy equation assumptions : (0.25pt)

-Steady state.
-One dimensional .
- With heat generated.

Cartesian coordinates

$$\nabla^2 T + \frac{q}{\lambda} = \frac{\delta^2 T}{\partial x^2} + \frac{q}{\lambda} = 0 \qquad \text{(0.25pt)}$$

Cylindrical coordinates

$$\nabla^2 T + \frac{q}{\lambda} = \frac{\delta^2 T}{\delta r^2} + \frac{1}{r}\frac{\delta T}{\delta r} + \frac{q}{\lambda} = 0 \qquad \text{(0.25pt)}$$

2) Laplace equation

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{q}{\lambda} \text{ (0.25pt)}$$

Energy equation assumptions : (0.25pt)

-Steady state.
-One dimensional .
- Without heat generated.

Cartesian coordinates

$$\nabla^2 T = \frac{\delta^2 T}{\partial x^2} = 0 \text{ (0.25pt)}$$

Cylindrical coordinates

$$\nabla^2 T = \frac{\delta^2 T}{\delta r^2} + \frac{1}{r}\frac{\delta T}{\delta r} = 0 \qquad \text{(0.25pt)}$$

2) Importance of fins in the industry.

- C'est une solution économique (moins coûteuse) consiste à augmenter la surface d'échange du solide, ce qui permet de réduire les pertes de chaleur. **(1.5 pt)**

3) List the criteria for choosing a fin. **(1.5 pt)**
Bonne conductivité thermique – Grande efficacité.

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Part 2 (15pts)

Problem N°1(5pts)

1) Calcul of Tmax

-Steady state /One dimensional (Axis X) / With heat generated. **(0.25pt)**
The passage of electric current produces heat, therefore Qp ≠0 (q ≠ 0).

The heat equation in cylindrical coordinates

$$\frac{\delta^2 T}{\delta r^2}+\frac{1}{r}\frac{\delta T}{\delta r}+\frac{q}{\lambda}=0 \Rightarrow r\frac{\delta^2 T}{\delta r^2}+\frac{\delta T}{\delta r}=-\frac{q}{\lambda}r \Rightarrow \frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{q}{\lambda}r$$

$$\Rightarrow \int \frac{d}{dr}\left(r\frac{dT}{dr}\right)=\int -\frac{q}{\lambda}r \Rightarrow \left(r\frac{dT}{dr}\right)=-\frac{q}{2\lambda}r^2+A \Rightarrow \left(\frac{dT}{dr}\right)=-\frac{q}{2\lambda}r+\frac{A}{r}$$

$$\Rightarrow \mathrm{T(r)}=-\frac{q}{4\lambda}r^2+A\,ln(r)+b$$ **(0.75pt)**

Conditions limites (r = 0⇒ A=0)

r = 0 ; T = T1 $\Rightarrow \mathrm{T1}=-\cancel{\frac{q}{4\lambda}r^2}+A\,ln(0)+b \Rightarrow \mathrm{A}=\frac{T1-b}{\ln(0)} \Rightarrow \mathrm{A}=0$ **(0.25 pt)**

r = R ; $T(r)=T2=-\frac{q}{4\lambda}r^2+b \Rightarrow b=T2+\frac{q}{4\lambda}R^2$ **(0.25 pt)**

$$\boldsymbol{T(r)=-\frac{q}{4\lambda}r^2+\frac{q}{4\lambda}R^2+T_2}$$ **(0.5 pt)**

The temperature is maximum $\Rightarrow \frac{dT}{dr}=0$ **(0.25 pt)** $\Rightarrow -\frac{q}{2\lambda}r=0 \Rightarrow r=0$ **(0.25 pt)**

$\boldsymbol{Tmax=\frac{q}{4\lambda}R^2+T_2}$ **(0.25 pt)** **with** $q=\frac{p}{V}=\frac{p}{S.L} \Rightarrow Tmax=\frac{R^2}{4\lambda}\frac{p}{S.L}+T_2$; **R** = $\sqrt{\frac{S}{\pi}}$

$$\textbf{N.A : } Tmax=\frac{0.0796}{4(0.85)}\frac{800}{(0.25).1}+120=194.92$$

$$Tmax=194.92\ \mathrm{C}$$ **(0.5 pt)**

2)Calcul of h

$$\emptyset cond=\emptyset Conv$$ **(0.25 pt)**

$\emptyset=-\lambda\,S\frac{dT}{dr}=hs(T2-T_f)$ **(0.25 pt)**; $\left(\frac{dT}{dr}\right)=-\frac{q}{2\lambda}r+\frac{A}{r}$ **(0.25 pt)** *with* $A=0$ & r = R

$\Rightarrow \emptyset=-\lambda\,S(-\frac{q}{2\lambda}R)=hs(T2-T_f) \Rightarrow (\frac{q}{2}R)=h(T2-T_f) \Rightarrow$ **h** = $\frac{q}{2(T2-T_f)}R$ **(0.5 pt)**

N.A : ⇒ **h** = $\frac{3200}{2(120-20)}0.282=4.5\ w/m^2C \Rightarrow$ **h**= $4.5\ w/m^2C$ **(0.5 pt)**

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Problem N°2 (6 pts)

Convection forcée, géométrie plaque :

T_{ref} = (120+30) /2 = 75°C **(1pt) / Ecoulement externe avec transfert de chaleur**
Gas properties (interpolation)

λ=0.201 (W/mC) $\nu = 140.3\ 10^{-6}(\frac{m^2}{s})$, Pr= 0.707 (0.6 < Pr < 50).

$Re = \frac{V.L}{\nu}$ **(0.25 pt)** $= \frac{5(3)}{140.3\ 10^{-6}}$ = 1.07 $10^5 < 5.10^5$ Régime laminaire **(1.75 pt)**

$\overline{NU} = \frac{h.L}{\lambda} = 0.664.Re^{0.5}.Pr^{0.33}$ = 193.74 **(1 pt)**

$h = \frac{\overline{NU_L}}{L}.\lambda$ **(0.25 pt)** = 12.98 (w/m²K) **(0.75 pt)**

$\phi = 2.h.S(Ti - T_\infty)$ (02 surfaces) **(0.25 pt)**

$\phi = 2.h.(3x1)(120 - 30)$

$\phi = 7009.2\ W$ **(0.75 pt)**

Problem N°3 (4pts)

Calcul of surface :

S $=4\pi r^2$ ⇒ 4π (D/2)² ⇒ S= π D² = 3.14 (1 392 000 .10³)² = 6.084 10^{18} m² **(1pt)**

Calcul of Sun heat flux

$\emptyset = \varepsilon\sigma sT^4$ **(0.5pt)** ⇒ $\emptyset = \sigma sT^4$; $\varepsilon = 1.$ σ = 5,67×10^{-8} SI.

N.A : $\emptyset = 5.67\times10^{-8}$ 6.084 10^{18} (5000)⁴ = 2.156 10^{26} w **(1pt)**

Calcul of heat flux (Sun-Eart)

$\varphi = \varepsilon\sigma(T_1^4 - T_2^4)$ (w/m²) **(0.5pt)** ; N.A : $\varphi = 0.75\ (5.67\times10^{-8})(5000^4 - 2773.15^4)$

$\varphi = 2.406\ 10^7$ w/m² **(1pt)**

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